AERODYNAMICS

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If we consider the ideal case of air streaming frictionless through a tube of constant diameter, then the pressure is the same everywhere along the tube.

Is the diameter variable then we find that Bernoulli's Law holds: $p + \frac{1}{2} \varrho v^2 = C$(constant), for every cross-section of the tube.

In this formula p is the static pressure and $\frac{1}{2} \varrho v^2$ is the aerodynamical pressure with ϱ the coefficient of density and v the velocity.

This means that when the air is flowing from a wider part of the tube into a narrower part, its velocity increases but the pressure falls.

In reality there is friction caused by the wall of the tube and the viscosity of the air and so there will be a pressure fall along the tube of constant diameter. In the case of a tube with variable bore we get a pressure fall in a narrower part, which is the sum of the fall due to Bernoulli's Law and the fall due to the friction.

Let we now consider the air-flow through a tube and recognize the friction. If the velocity is not too high the flow will be laminar, i.e. in a round tube the velocity is the highest in the axis and decreases in the direction of the walls. The velocity distribution can be given in the formula:

$v_r = 2 \bar{v} (1 - \frac{r^2}{R^2})$, in which v_r is the velocity at distance r from the axis, R is the radius of the tube ($0 \leq r \leq R$) and $\bar{v}$ is the mean velocity (fig. 1Aa).

The pressure fall dp over a small distance dl along the tube is given by Poisseuille's Law : $\frac{dp}{dl} = - \frac{8 \eta V}{\pi R^4}$, in which V is the volume discharged per second and η the coefficient of viscosity.

If the velocity increases laminar flow is changing in turbulent flow, i.e. the velocity at a distance r from the axis is not constant anymore as well in magnitude as in direction and at some places vortices occur (fig. 1Ab). If the velocity still more increases, then the whole flow becomes turbulent (fig. 1Ac). We have now a different formula for the pressure-fall along the tube:

$$\frac{dp}{dl} = - 9.10^{-3} \frac{\nu^{1/4} \varrho V^{1,75}}{R^{4,75}},$$

inwhich ν is the kinematical viscosity $\frac{\eta}{\varrho}$. From this follows, that in laminar flow the pressure fall is proportional to the discharged volume per second V and in turbulent flow increases to proportional to $V^{1,75}$. In between is a range in which the flow is partially laminar, partially turbulent (fig. 1Ba, b and c). Even in turbulent flow there is close to the wall still laminar flow.

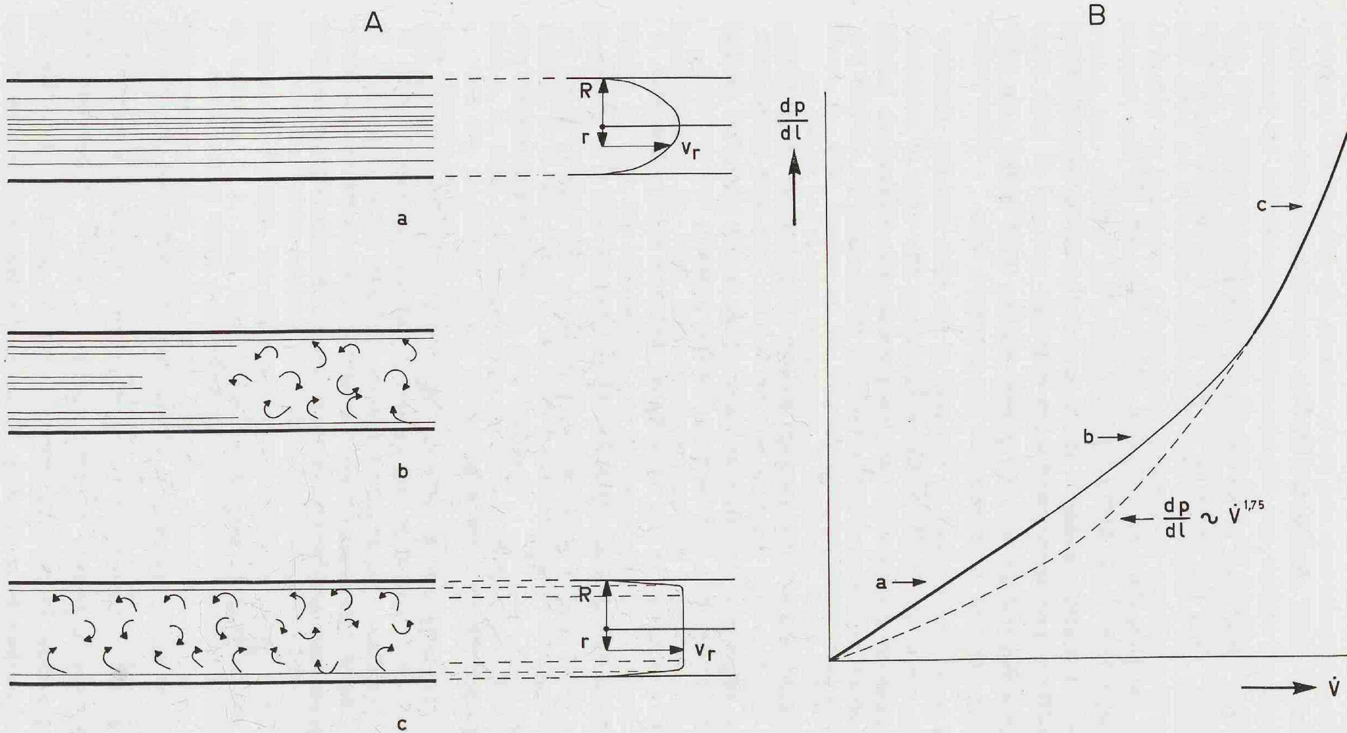

Fig. 1 A: Relation between velocity and type of flow:
a) Laminar flow,
b) Partly laminar - partly turbulent flow,
c) Turbulent flow.

1 B: Diagram giving the relation between pressure-fall $\frac{dp}{dl}$ and discharged volume per second V in the cases a, b and c.

Fig. 1 A: Relation entre la vitesse et le type d'écoulement.
a) Régime d'écoulement laminair.
b) Régime d'écoulement partiellement laminair et partiellement turbulent.
c) Régime d'écoulement turbulent.

Fig. 1 B: Graphique de la relation entre la perte de charge $\frac{dp}{dl}$ et le débit V dans les cas a, b, c.

When is the flow laminar and when turbulent? This is determined by Reynolds' number. This is a number of zero dimension : $Re = \frac{2V}{\pi \nu R}$. The critical value for smooth round tubes is $Re_{cr} = 2200$. When $Re > 2200$ the flow is turbulent and for $Re > 2200$ laminar.

With the knowledge of Reynolds' number we can calculate for a certain value of R the critical value of the volume per second V_{cr} or for a certain value of V we can calculate the critical radius.

Example: the air-flow through the trachea with $R = 1.0$ cm, $\nu = 0{,}149$ cm^2 sec^{-1} and $V = 250$ cm^3 sec^{-1} gives $Re = 1000$. This flow will certainly be laminar. If the radius decreases to less than half this value then there is a possibility for turbulence and in the nose we always may expect turbulence somewhere.

With the help of Reynolds' number we can transfer results found in experiments on flow to other situations provided Re remains the same. E.g. the results of experiments with water can be transferred to air if the velocity of the water (discharged volume per second) is 14 times smaller than the air velocity, because $\nu_{water} = 0{,}0106$ and $\nu_{air} = 0{,}149$ cm^2 sec^{-1} (at 18° C).

In addition to the flow through a tube we have to consider the in-flow and out-flow phenomena.

During in-flow we expect a pressure fall of $\frac{1}{2} \varrho \overline{v}^2$ due to the gain of kinetic energiy, but this changes into $\varrho \overline{v}^2$ because of the Poiseuille flow and additional friction losses in the beginning bring it up to $1{,}12\ \varrho \overline{v}^2$.

During out-flow, in the ideal case, the loss of velocity should as a whole be transformed in pressure (Bernoulli) : $p_2 = p_1 + \frac{1}{2} \varrho (v_1^2 - v_2^2)$.

Part of the kinetic energy, however, is transformed in heat because of collision losses during release of the flow. Because of this the pressure raises to $p_2 = p_1 + \varrho v_2 (v_1 - v_2)$. Suppose $v_2 = \alpha v_1$, in which $0 < \alpha < 1$, then $p_2 - p_1 = \frac{1}{2} \varrho v_1^2 . 2 \alpha (1 - \alpha) = C v_1^2$ (C = constant).

Summarizing we can say that during respiration through the nose we will get in the formula for the pressure difference over the nose terms with V (laminar flow), $V^{1,75}$ (turbulent flow) and V^2 (in- and outflow).

Thus: inspiration $P_{nose} = - C_1 V - C_2 V^2 - C_3 V^{1,75}$

expiration $P_{nose} = - C'_1 V + C'_2 V^2 - C'_3 V^{1,75}$

The constants C and C' will not only differ each from another but also differ from person to person and from moment to moment.

SUMMARY

In the ideal case of air-flow through a tube without friction Bernoulli's Law holds.

Is there any friction then is the flow laminar or turbulent. This is determined by Reynolds' number. The critical value is about 2200 and turbulence occurs if this value is exceeded. In experiments it is permissible to transfer results obtained in one situation to another if Reynolds' number remains the same,

e.g. experiments with water and air. It is necessary to recognize the in-flow and out-flow phenomena of a tube.

If the volume discharged per second is V, then the pressure-fall in the nose contains terms with V (laminar flow), $V^{1,75}$ (turbulent flow) and V^2 (in- and out-flow).

AÉRODYNAMIQUE

La loi de Bernoulli est valable dans le cas idéal où l'air s'écoule sans frottement à travers un tube. Mais s'il y a frottement, l'écoulement peut être laminair ou turbulent. La détermination en est faite par le nombre de Reynolds. La valeur critique est approximativement 2200, et en dépassant cette valeur l'écoulement devient turbulent.

Dans les essais concernant l'écoulement, on peut reporter les résultats obtenus dans une situation à une autre situation, à condition que le nombre de Reynold 5 reste le même, par exemple, dans des essais avec l'air et l'eau.

De plus il est nécessaire de tenir compte des phénomènes d'écoulement intérieur et extérieur du tube.

Si le débit est V, la formule de la perte de charge dans le nez comprend des termes avec V (écoulement laminair), $V^{1,75}$ (écoulement turbulent) et V^2 (écoulement intérieur et extérieur).

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