Modelling nasal airflow coefficients: an insight into the nature of airflow*

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Abstract

Background: There has been considerable discussion and conflicting views regarding the presence of laminar or turbulent flow within the nose. The aim of this study was to investigate how the modelling of variable flow coefficients can assist in the evaluation of the characteristics of flow in the resistive segments of the nose.

Methodology: A comparison was made between the flow coefficient for the nasal valve, obtained from a mathematical model, and resistive flow components such as a Venturi meter and orifice tube. Also, a variable loss coefficient was formulated for the whole (unilateral) nose which, by utilising the intersection of the laminar and turbulent asymptotes, provided an estimation for the critical Reynolds number ($Re_{crit}$).

Results: The results show that the flow resistance of the nasal valve is considerably greater than that for both a Venturi meter and an orifice tube implying turbulent or turbulent-like flow for much of nasal inspiration. Regarding the loss coefficient for the whole (unilateral) nose, normal respiration flowrates are displaced well away from the laminar asymptote. The critical Reynolds number was estimated to be 450.

Conclusions: A novel method of determining the flow characteristics of the nose, particularly the critical Reynolds number, is presented. The analysis indicates a higher degree of turbulence than is assumed from a simple traditional calculation using a hydraulic diameter and flow through straight tubes. There are implications for computational fluid dynamics (CFD) modelling where either the entire nasal airflow is assumed to be laminar or a low turbulence model implemented.

Key words: computational fluid dynamics, critical Reynolds number, nasal modelling, nasal valve, unsteady flow

Introduction

Given the air conditioning function of the nose, the inspiratory airflow route through the nose and the degree of turbulence are important considerations. For example, abnormal flow has been associated with atrophic rhinitis, empty nose syndrome, turbino-plasty and rhinosinusitis with nasal polyposis (36, 55-57). Regarding

Nomenclature: A: Cross sectional flow area; $C_f$: Flow coefficient (discharge coefficient) = actual flow rate/theoretical flow rate; $K_{loss,turbinate}$: Loss coefficient for the turbinate region; $K_{loss,nose}$: Loss coefficient for the whole (unilateral) nose; L, k, srt: Constants associated with the Logistic function used to model the discharge coefficient $C_f$ for the nasal valve (Table 1). $Δp$: Transnasal pressure drop; Q: Volumetric flow rate; Re: Reynolds number = ratio of inertial forces to viscous forces; $Re_{crit}$: Critical Reynolds number (Reynolds number associated with laminar/turbulent transition); $ST$, $mag$, $Adj$: Constants associated with the tanh function used to model the loss coefficient for the turbinate region ($K_{loss,turbinate}$); $u$: Mean fluid velocity; $p$: Density; $v$: Kinematic viscosity; $μ$: Dynamic viscosity; λ, ξ: Coefficients associated with the linear and non-linear terms of the polynomial curve fit of the rhinomanometric data for the whole (unilateral) nose (equation 1). Whilst an explanation of some fluid mechanics terms has been incorporated into the Introduction, some readers may desire a more detailed coverage of elementary theory. If so, we recommend the first half of reference 54.
Nasal flow coefficients

A theoretical analysis of the nasal pressure -v- flow data, there have been several mathematical models which have utilised Bernoulli’s equation with the incorporation of a discharge coefficient, similar to the treatment of flow through a Venturi meter or orifice restriction (1-3). The discharge coefficient (Cd) is simply a measure of how efficiently a flow component passes a given ‘real’ fluid compared to an ‘ideal’ fluid; the latter being incompressible and having no viscosity. Since the discharge coefficient is the ratio of the ‘actual’ to the ‘theoretical’ flowrate, then an efficient flow component, such as a Venturi meter, has a discharge coefficient close to 1.0 and a relatively small pressure loss compared to a less efficient component such as an orifice plate (Figure 1). However, for the nose, a significant pressure loss is not necessarily an indication of inefficiency since it is associated with increased mixing of the inspiratory airstream which aids air conditioning. Arguably, over-zealous surgery which results in a very low transnasal pressure drop may not be the most optimal solution for the patient.

The fact that the flow coefficient is dependent upon the prevailing Reynolds number, particularly at low flowrates, has received limited attention. The Reynolds number (Re) is dimensionless being the ratio of inertial forces to viscous forces. It is directly proportional to the fluid velocity such that a lower flowrate will result in a lower Reynolds number, reflecting an increasing influence of viscous forces. For steady flow in long straight pipes, at Reynolds numbers below about 2,000 viscous forces become increasingly important. The effect is to inhibit turbulence with the flow transitioning to become more laminar in character. For complex geometries there is more disruption of the flow such that turbulence may be present, say at a Reynolds number of 500, which would not be predicted from a simple pipe flow analogy. From measurements in a plastinated nose model, Fodil et al. (4) used a pressure-drop (loss coefficient) adjustment proportional to Re
0.571 for Re<390 albeit they seem to attribute no physical significance to this requirement. Also, in a computational fluid dynamics (CFD) simulation, Zamankhan et al. (5) provide an equation for a variable friction factor for Re<500 which can be shown to be proportional to Re
0.75. More recently, and using separate variable coefficients for the nasal valve and turbinate region, O’Neill and Tolley (6) were able to show a much better fit to experimental results (rhinomanometry) than for the case where the coefficients were held constant.

Figure 1 shows typical calibration curves for Venturi meter, orifice tube (cylindrical choke) and orifice plate. For specific data refer to references (7-15). The large variation in the discharge coefficient at low Reynolds numbers for each device is clearly evident. Industrial flowmeters typically operate at high Reynolds numbers (Re >> 10^3) where the discharge coefficient (Cd) remains reasonably constant. In contrast, the Reynolds number during nasal breathing varies continuously within an approximate range of 0<Re<2×10^3. Assuming some degree of similarity between the characteristics of the above devices and the nose, the variation in the flow coefficient(s) throughout the nasal breathing cycle is an important consideration.

Whilst it may be tempting to apply the data of Figure 1 directly to nasal airflow, there are additional influences at play which indicate that a more accurate relationship, specific to the nose, is required. Firstly, flowmeters are calibrated during steady-state operation (constant flowrate) and secondly, the meter is located in a long length of straight pipe which approximates to fully-de-
developed flow conditions upstream. Neither of these conditions is met for the case of airflow through the major resistive segments within the nose. Because of the more complex flow conditions, it might well be argued that the flow characteristics for the nose, whilst reflecting the effects of transitional flow within the range of low to moderate Reynolds numbers, should deviate somewhat from those for a flowmeter. In the material which follows we will endeavour to show that this is indeed the case and, in particular, illustrate how the variable discharge coefficient for the nasal valve (and the variable loss coefficient for the turbinate region) can be modelled by the use of the Logistic (and tanh) function. In addition, a loss coefficient for the whole (unilateral) nose is presented which is used to provide an approximation to the critical Reynolds number \( R_{\text{crit}} \).

**Materials and methods**

For the nasal valve

For the purpose of this study existing information for flow through Venturi, orifice-plate and orifice-tube devices was used as a comparison to the flow characteristics for the nasal valve; the latter being obtained from the mathematical model of O’Neill and Tolley \(^6\). The studies used for this comparison are listed in Table 1 together with analytical expressions for the discharge coefficient \( v \)- Reynolds number. For the valve the expression is a modified form of the Logistic function. This, we found, provided a more flexible and better fit to the rhinomano-metric experimental data than was the case for alternatives such as the 2K, two-coefficients method described by Hooper \(^{16}\). For each device in Table 1, the expression for the discharge coefficient was differentiated with respect to the Reynolds number in order to evaluate the likely prevailing flow conditions between mainly laminar (where the slope \( d(C_d)/dR_e \) has a high value) and turbulent (where the slope \( d(C_d)/dR_e \) is close to zero). This may be explained as follows:

As shown by Johansen \(^7\) and others \(^{11,17}\), studying orifice flow at very low Reynolds numbers \( R<10 \) the discharge coefficient is proportional to the square root of the Reynolds number i.e. \( C_d \propto R^{0.5} \), and so the differential \( d(C_d)/dR_e \) \( \propto R^{-0.5} \). Thus, as \( R \rightarrow 0 \), slope \( \rightarrow \infty \). At high Reynolds numbers \( C_d \) is approximately constant, and so \( d(C_d)/dR_e \approx 0 \). Thus, over the range of low to high Reynolds numbers, the magnitude of the slope \( d(C_d)/dR_e \) varies from a high value to a value close to zero. Comparing this relationship for the nasal valve to those for the flow devices listed above gives some insight of the flow conditions prevailing for the valve.

\[ C_d = \frac{1}{(1 + 5.35 \sigma^{-0.5})} \]

where \( \sigma = 0.89 \times \frac{R}{(\text{Diameter}/\text{Length})} \)

\[ C_d = \frac{0.995}{1 + 192 / (\text{Re})^{1.5}} \]

\[ C_d = \frac{L}{1 + \exp(-k \times 10^{-3}R_e)} - \text{srt} \]

where \( L = 0.68; k = 0.55 \) and \( \text{srt} = 0.1 \)

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\*The expression is often referred to as Rohrer’s equation \(^{18}\). However, the equation (in slightly different forms) has a longer history than most researchers might be aware. Gaspard Prony’s polynomial equation was used by 19th century hydraulic engineers in the design of city water supplies. Henry Darcy improved upon this with a publication which accommodated the effect of pipe roughness \(^{19}\).
Nasal flow coefficients

For the turbinate region

Whereas a coefficient of discharge \( C_d \) is commonly used for Venturi and orifice flow, we modelled the flow-v-pressure relationship of the turbinate region by an analogy with ‘minor’ (local) losses in pipe flow. Here, \( K_{turbinate} \) is an empirically determined loss coefficient used to account for the pressure drop from the complex flow through the localised resistance. As described previously, we first modelled a coefficient of discharge for the turbinate region \( (C_{d\_turbinate}) \) by a method of trial and error using the tanh function. This was then converted to a loss coefficient \( K_{turbinate} \) using the relationship between the flow through an orifice (or orifice tube) and the Darcy-Weisbach equation applied to the case of minor losses such that

\[
K_{turbinate} = (C_{d\_turbinate})^2
\]

The relationship is:

\[
K_{turbinate} = [ST + \{mag \times \tanh(R_0^{0.5} \times Adj)\}]^{-2}
\]

where \( ST = 0.043; \) \( mag = 0.20 \) and \( Adj = 0.029. \)

For the whole (unilateral) nose

Representing the rhinomanometric flowrate-v-pressure difference by a second degree polynomial equation of the form

\[
\Delta p = \lambda \dot{Q} + \xi \dot{Q}^2
\]

and the equation for a local pressure loss by

\[
\Delta p = K_{L\_nose} \rho \frac{u^2}{2}
\]

The continuity equation is

\[
\dot{Q} = uA
\]

and Reynolds number \( Re = \frac{\rho u A^{1/2}}{\mu} \)

where the square root of the cross-sectional flow area is used as the hydraulic diameter.\(^{19,20,21}\)

Equating (1) and (2) and utilizing the relationships in (3) and (4) gives the loss coefficient:

\[
K_{L\_nose} = \frac{2A^{3/2}}{\mu Re} + \frac{\xi 2A^2}{\rho}
\]

The relationship is the sum of two asymptotes. At high Reynolds numbers the first term on the right-hand side is negligible and the second (constant) term represents the inertial-turbulent asymptote. At low Reynolds numbers the first term predominates representing the (variable) viscous-laminar asymptote. Some researchers involved with the analysis of flow through valves and similar resistive components consider the intersection of the two asymptotes to be the location of the critical Reynolds number i.e. the region of laminar-turbulent transition. Others consider this estimate to be too high and use a value based upon the initial departure from the laminar asymptote.\(^{22}\). The value for the critical Reynolds number can also be obtained directly from Eqn (5) by equating the two asymptotes and rearranging so that

\[
Re_{crit} = \frac{\lambda}{\xi} \left( \frac{1}{v A^{0.5}} \right)
\]

where \( v = \frac{\mu}{\rho} \)
Results

Nasal valve

In Figure 2A it is evident that the flow resistance of the nasal valve is considerably greater than that for both a Venturi meter and an orifice tube. For the valve, the discharge coefficient at a Reynolds number of 1,000 is about 0.33 compared to about 0.74 for an orifice tube and 0.9 for a Venturi meter. Figure 2B provides more of an insight of the flow conditions prevailing across the range of Reynolds numbers. At Re > 1,000 the low slopes of all the devices and nasal valve indicate a turbulent flow regime. For Re < 1,000 the slopes for the devices increase significantly at decreasing Reynolds numbers, consistent with reducing turbulence. However, the low slope for the nasal valve extends well below a Reynolds number of 500, implying significant turbulence or at least turbulent-like flow for much of nasal inspiration.

Turbinate region

Figure 3 shows the relationship between the turbinate region loss coefficient and the associated Reynolds number. The flow area used in the calculations is that of the turbinate region (144 mm²) since, unlike industrial meter applications, there is no distal upstream (usually smaller) flow area to use as a reference. One possibility is the nasal valve dimension which, if used, would move the KL-turbinate curve vertically downwards.

Whole (unilateral) nose

Figure 4 shows rhinomanometry flow rate – v – pressure difference data and a polynomial curve fit as described by Eqn (1) [Methods]. When Δp and Q̇ are expressed in units of Pa and m³s⁻¹, respectively, then λ = 85.2×10³ and ξ = 1.28×10⁹. Note that flowrate is plotted on the abscissa and both axes are logarithmic. The graph has been extended well beyond the normal physiological range simply to illustrate the relationship of the fitted polynomial to the turbulent asymptote. Nasal resistance (Δp/ Q̇) is usually calculated using a reference pressure difference of 150 Pa (sometimes 75 Pa). The location for the region of major flow during inspiration (shaded oval) is based upon a respiration minute volume of 3 Lt. min⁻¹ (one cavity) and a respiration rate of 12 min⁻¹. As the graph illustrates, reference pressures and normal respiration flowrates are displaced well away from the laminar asymptote. The relationship between the loss coefficient and Reynolds number as evaluated according to Eqn (5) is shown graphically in Figure 5. The flow area used in the calculations is 100 mm² which is a compromise between that for the turbinate region (~140 mm²) and that for the nasal

<table>
<thead>
<tr>
<th>Flow area A (mm²)</th>
<th>Critical Reynolds no. Rcrit</th>
</tr>
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<tbody>
<tr>
<td>60</td>
<td>573</td>
</tr>
<tr>
<td>100</td>
<td>444</td>
</tr>
<tr>
<td>140</td>
<td>375</td>
</tr>
</tbody>
</table>
Nasal flow coefficients

\[ \frac{\rho}{v} = \frac{\rho R}{\nu} \]

estimated to be 450 – comparable to Recrit reported for a range of about 100<Re<1,000 (23-25). In this present study, the localised lesions (stenosis/aneurysm), which disrupt the flow, significantly reduce the critical Reynolds number to within the range of about 100<Re<1,000 (23-25). In this present study, the critical Reynolds number for the whole (unilateral) nose was estimated to be 450 – comparable to Re_m reported for a range of valves, orifices and nozzles (26-28). Interestingly, Schreck et al. (29), experimenting with a 3:1 scale physical nasal model, report a transition to turbulence at a Reynolds number of approximately 600. Examination of the experimental results in the 1:1 scale physical model of Croce et al. (30) indicates a similar result. Also, Sullivan and Chang (31) obtained rhinomanometric data from five healthy adults subjected to an externally controlled flow. Their Figure 3 shows the dimensional pressure loss as a function of the Reynolds number. A comparison of these experimental results with the analysis from this present study is shown in Figure 6. The considerable difference in the vertical displacement between the studies can be explained largely by the different reference dimensions used in the flow calculations. Whereas Croce et al. (30) used a diameter of 2.48 cms taken from the connection tube, Sullivan and Chang (31) used a hydraulic diameter (8 mm - median) based upon the area of the nostril. Regarding the shapes of the curves there is considerable similarity to the loss coefficient obtained from this present study which is based upon a meta-analysis of rhinomanometric data. Arguably, this represents the most accurate nasal loss coefficient data to date. During inspiration, the disturbance of flow through the nasal valve constitutes a proximate upstream disturbance to the flow through the turbinate region. Thus, although the turbinate region's Reynolds number is well within the limits for laminar flow, the shapes of the curves there is considerable similarity to the loss coefficient obtained from this present study which is based upon a meta-analysis of rhinomanometric data. Arguably, this represents the most accurate nasal loss coefficient data to date.

### Discussion

Within the nose there are two distinct resistive components – the nasal valve region, situated anteriorly with a minimum cross-sectional area of about 60 mm² and the main nasal cavity (turbinate region) with a flow area of about 140 mm². These dimensions result in a Reynolds number, during quiet breathing, reaching about 1000 for the valve and 600 for the turbinate region. This study has shown that the flow resistance of the nasal valve is significantly greater than that for orifice flow for which Johansen (27) reports significant turbulence observable at a Reynolds number of 1,200. Also, regarding physiological flows in otherwise normal vasculature, there is evidence that localised lesions (stenosis/aneurysm), which disrupt the flow, significantly reduce the critical Reynolds number to within the range of about 100<Re<1,000 (23-25). In this present study, the critical Reynolds number for the whole (unilateral) nose was estimated to be 450 – comparable to Re_m reported for a range of valves, orifices and nozzles (26-28). Interestingly, Schreck et al. (29), experimenting with a 3:1 scale physical nasal model, report a transition to turbulence at a Reynolds number of approximately 600. Examination of the experimental results in the 1:1 scale physical model of Croce et al. (30) indicates a similar result. Also, Sullivan and Chang (31) obtained rhinomanometric data from five healthy adults subjected to an externally controlled flow. Their Figure 3 shows the dimensional pressure loss as a function of the Reynolds number. A comparison of these experimental results with the analysis from this present study is shown in Figure 6. The considerable difference in the vertical displacement between the studies can be explained largely by the different reference dimensions used in the flow calculations. Whereas Croce et al. (30) used a diameter of 2.48 cms taken from the connection tube, Sullivan and Chang (31) used a hydraulic diameter (8 mm - median) based upon the area of the nostril. Regarding the shapes of the curves there is considerable similarity to the loss coefficient obtained from this present study which is based upon a meta-analysis of rhinomanometric data. Arguably, this represents the most accurate nasal loss coefficient data to date.

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Table 3. Cross-sectional area of the nasal valve from different studies. Values reported for both nasal cavities have been halved for comparison purposes. CT, computerised tomography; MRI, magnetic resonance imaging; AR, acoustic rhinometry.

<table>
<thead>
<tr>
<th>Method</th>
<th>Wen et al. (42)</th>
<th>Subramaniam et al. (51)</th>
<th>Cheng et al. (52)</th>
<th>Keyhani et al. (53)</th>
<th>Garcia et al. (56)</th>
<th>O’Neill and Tolley * meta-analysis (single cavity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valve area (mm²)</td>
<td>CT 70</td>
<td>MRI 80</td>
<td>MRI 95</td>
<td>CT 102</td>
<td>MRI 110</td>
<td>AR decongested 61</td>
</tr>
</tbody>
</table>

maximum (32,33). Given that normal nasal breathing is almost effortless and provides the required mass flow at the appropriate temperature and humidity for the lower airway, it is justifiable to view the process as an example of system optimisation. Here, there is an inverse relationship between thermal resistance and flow resistance (respiratory effort) (34,35). An important parameter regarding the effectiveness of heat and mass transfer processes is the ‘surface area density’ (β or SA:V) which is the ratio of surface area to volume. When expressed in units of m² and m³, the value for the nose is about 1,000 (36) which compares well with the definition of a compact heat exchanger where β > 700 (37). For the human lung β is ∼20,000 (38). Undoubtedly, the nature of flow through the nasal valve region plays a vital role in this heat and mass transfer process.

There are clear implications for computational fluid dynamics (CFD) modelling where either the entire nasal airflow is assumed to be laminar or a low turbulence model implemented (39). It is remarkable that the results from some turbulence models are not significantly different than those from a laminar flow model (40). However, CFD modelling often predicts a very low trans-nasal pressure drop and hence a significant difference in nasal resistance values compared to rhinomanometry even when allowance is made regarding the higher reference pressure in the latter (41-44). It is notable that cross-sectional areas for the nasal valve area in particular has been overestimated. It follows that the true extent of turbulence which is present in-vivo is not adequately represented. In addition, there are difficulties in solving the Navier-Stokes equations for complex flows, with Wong (46) suggesting that a statistical approach may be more appropriate. Whilst a degree of turbulence is advantageous for the air conditioning function of the nose it has the potential of posing considerable difficulty for analysis by computational fluid dynamics.

Given that there has been considerable discussion and conflicting views regarding the presence of laminar or turbulent flow within the nose with studies involving physical models, mathematical models and numerical methods (29,30,47-50), this study has provided a unique alternative method of analysis. In particular, the estimate of the critical Reynolds number can be obtained from rhinomanometric data and therefore has the potential for clinical use where abnormal flow is suspected such as in those cases listed at the beginning of the introduction.

**Conclusion**

Important insights into the characteristics of inspiratory nasal airflow have been gained by utilising variable flow coefficients for the nasal valve, turbinate region and the whole (unilateral) nose. The nasal valve promotes turbulent or turbulent-like flow which aids in effective mixing for air conditioning. The critical Reynolds number for the nose is about 450.

**Authorship contribution**

GO conceived and designed research; GO analyzed data; GO interpreted results of experiments; GO prepared figures; GO and NST drafted manuscript; NST edited and revised manuscript; GO and NST approved final version of manuscript.

**Conflict of interest**

No conflicts of interest are declared by the authors.

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