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The use of a mathematical model in rhinomanometry

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SUMMARY

The authors consider the mathematical model proposed by the Swedish Group (Broms et al.). This model permits the pressure gradient-flow recording as obtained from anterior or posterior rhinomanometry to be converted into a mathematical formula. The model was tested for its mathematical, statistical, and clinical utility with 32 normal test subjects. It is the conviction of the authors, although not totally without reservation, that this is the best mathematical model in existence.

INTRODUCTION

At the last Congress of the European Rhinologic Society in Davos (1978), there was a small separate symposium concerning Rhinomanometry.

It was universally accepted that the most accurate form at present is active rhinomanometry. The procedure utilizes no nozzles.

Simultaneous recording is made of the pressure gradient between the mask and the nasopharynx on one hand and the total air flow through the mask on the other hand.

This is not to say that other forms of rhinomanometry are unsatisfactory. The ultimate aim determines the method. Since the advantages and disadvantages of each method have been dealt with in other publications (Clement et al., 1978; Melon, 1979), they will not be further mentioned.

PROBLEM STATEMENT

The aim of any form of rhinomanometry is to provide a quantitative description of the function of each nasal cavity under all circumstances. Since the resistence of the nose is very flow dependent, it is advisable to vary the flow. In consequence, we are obliged to use active rhinomanometry in which a graph of Δp against $\mathring{V}$ is constructed. However, such a graph is difficult to use in a statistical analysis, therefore we utilized a mathematical model that approximated the real flow-pressure relation as closely as possible.

This model should not only be satisfactory from a statistical and mathematical viewpoint but should also be clinically usable. Therefore, the model must satisfy:

- mathematical norms
- statistical norms
- clinical norms.

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The mathematical model of the Swedish Group (Broms et al., 1979) satisfies these norms. It provides values for nose resistence in standardized conditions. It further provides values which are sufficiently normally distributed, thus allowing statistical analysis. The mathematical model permits the pressuregradient-flow recording to be constructed using easily calculated parameters. With these parameters for right and left nose side, the total nose resistence can be determined. To facilitate this, we have used a programmable calculator.

As one, for example, compares the curves from one nasal cavity, of different test subjects, the following features become apparent.

1. they run very smoothly
2. they are very similar
3. they cut a circle with a radius of 0.3 l/sec–3 m·Bar.

The curve of the right nasal cavity (Figure 1) crosses the flow axis at the origin with an angle v_0. The further from the origin, the greater the angle due to line curvature.

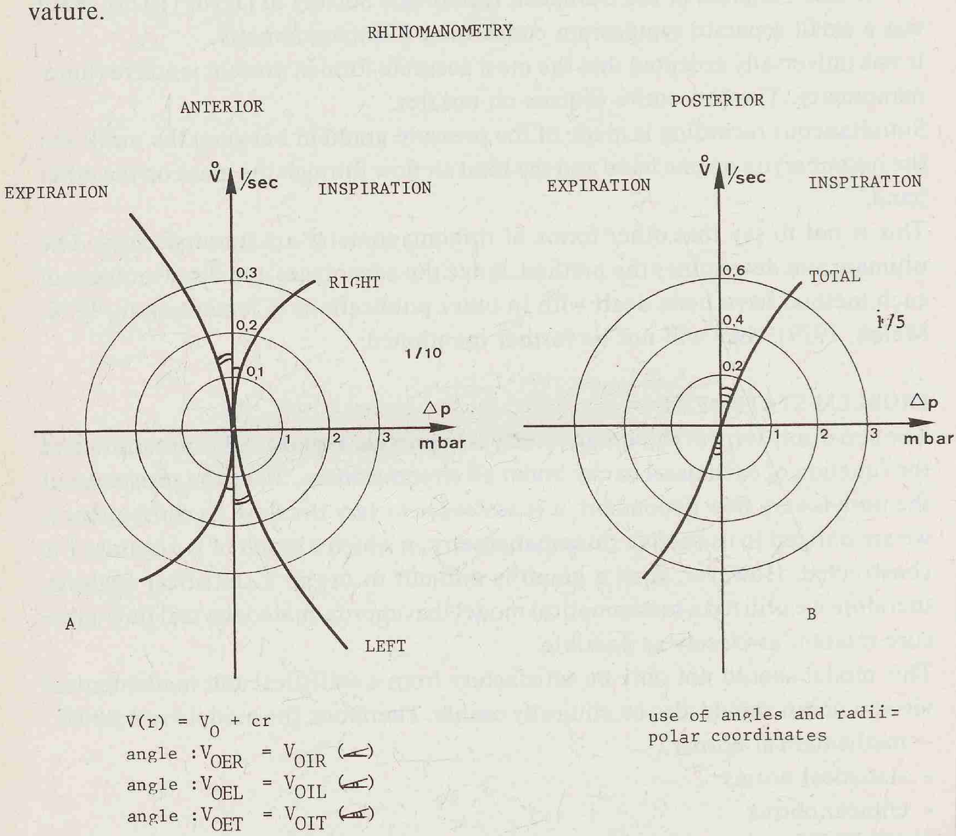


Figure 1. A. Anterior Rhinomanometry of right and left nasal cavity.
B. Posterior Rhinomanometry of the total nose in the same subject.

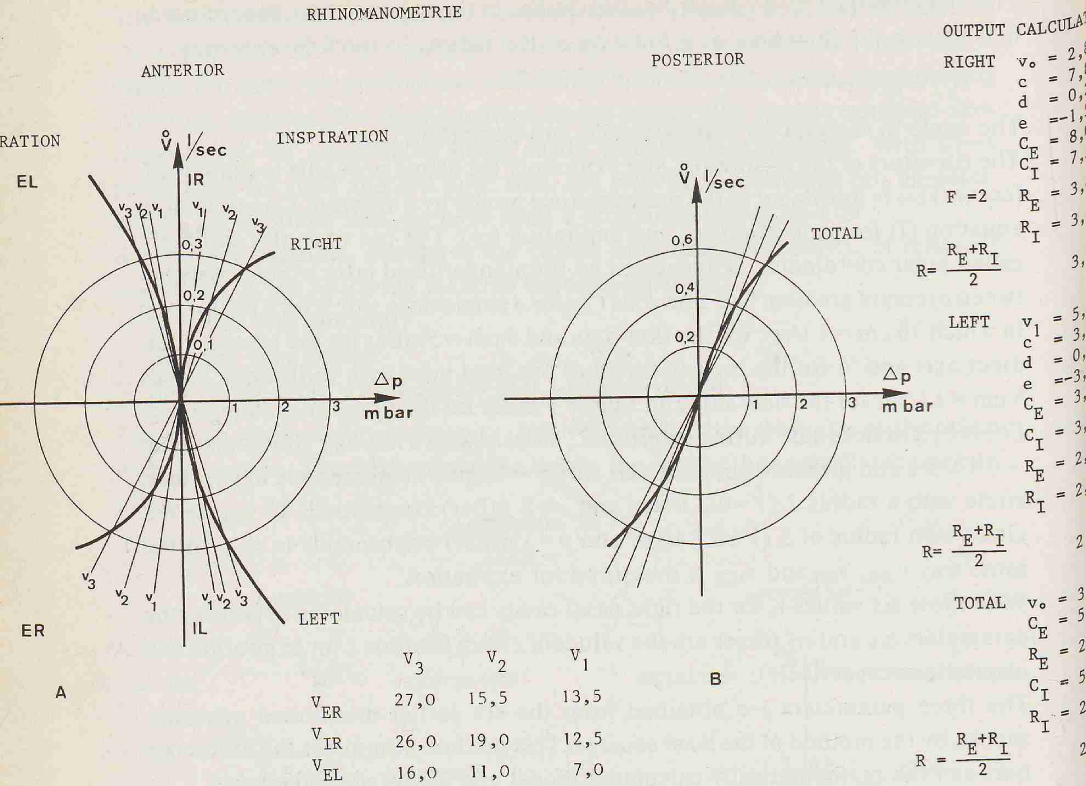
The mathematical model describes the change in the angle v which the recording makes with the flow axis, as a function of the radius, in the following way:

$$v(r) = v_0 + c \cdot r \qquad (1)$$

The angle v_0 is equal for inspiration (I) and expiration (E).
The curvature of the inspiratory and expiratory leg of one nose side is usually different. This is translated in the mathematical model by a different C parameter in equation (1) for inspiration (c_I) and expiration (c_E). The use of angles and radii, called polar coordinates, is suggested by the standardized ratio which exists between pressure gradient and flow axis ($^1/_{10}$ for a single nose side with a calibration in which 10 cm = 1 l/sec for the flow axis and 5 cm = 5mBar for the pressure gradient axis and $^1/_5$ for the measurement of the total nose with calibration values 5 cm = 1 l/sec for the flow axis and 5 cm = 5 mBar for the pressure gradient axis).
For the right nose side during inspiration, a circle with a radius 1 (flow axis value = 0.1 l/sec and pressure gradient axis value = 1 mBar) corresponds to v_{1IR}, for a circle with a radius 2 ($\mathring{V}$=0.2 l/sec. and p=2 mBar) corresponds to v_{2IR}, and a circle with radius of 3 ($\mathring{V}$=0.3 l/sec and p=3 mBar) corresponds to v_{3IR}. In the same way v_{1ER}, v_{2ER} and v_{3ER} is measured for expiration.
With these six values v_0 for the right nasal cavity can be calculated as well as the parameters c_{ER} and c_{IR} (these are the values of c from formula 1 for expiration and inspiration respectively).
The three parameters are obtained from the six earlier mentioned measurements, by the method of the least squares. This method minimizes the difference between the mathematically calculated model and actual measurements.
The Swedish authors found that the calculated values of the mathematical model at a specific flow and the actual values of the recording coincided very well, proving the utility of the model. Also they established that the use of more than six measurements (three for inspiration and three for expiration) yielded no significant improvement in adapting the mathematical model to the (real) recording. Therefore, six measurements are obtained for the recording of the right nasal cavity; upon inspiration v_{1IR}, v_{2IR}, v_{3IR} and upon expiration v_{1ER}, v_{2ER} and v_{3ER}.
From these, and using the mathematical model and a calculator, the values v_{OR}, c_{ER} and c_{IR} can be derived. An indentical procedure is then used for the left nasal side. In practice, six values are determined (Figure 2) to mathematically reconstruct the recording (Table 1):

Table 1.

right nasal cavity	r_3	r_2	r_1
expiration	$v_3 = 27.0$	$v_2 = 15.5$	$v_1 = 13.5$
inspiration	$v_3 = 26.0$	$v_2 = 19.0$	$v_1 = 12.5$



	V_3	V_2	V_1
V_{ER}	27,0	15,5	13,5
V_{IR}	26,0	19,0	12,5
V_{EL}	16,0	11,0	7,0
V_{IL}	17,0	14,0	13,0
V_{ET}	25,0	20,0	13,5
V_{IT}	29,0	20,0	12,5

Figure 2. A. Anterior Rhinomanometry of right and left nasal cavity.
B. Posterior Rhinomanometry of the total nose in the same subject.

Then one proceeds in the same way for the left nasal cavity. For the total nose, a difficulty is encountered, namely that for the same pressure gradient, the flow is twice as high due to respiration through both nasal orifices. As a consequence, the scale of the flow axis is doubled and the circle has been supplanted by an elipse.

Mathematically considered, an elipse is irrational and has to be avoided. If however, the flow scale is compressed by a factor of two, one again obtains a circle. In the case of the total nose, a circle radius of 2 mBar crosses the flow axis at 0.4 l/sec instead of 0.2 l/sec as is the case of one nasal cavity.

If one accepts that:

$$R = \frac{\Delta p}{\mathring{V}}$$

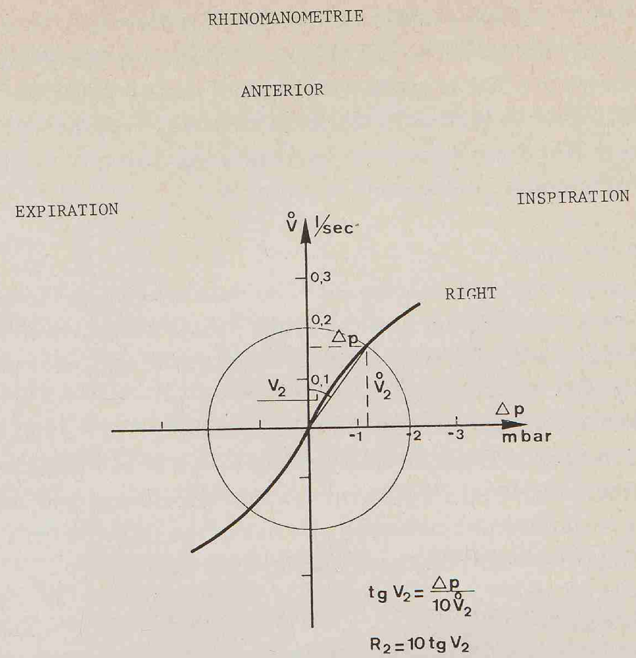


Figure 3. Anterior Rhinomanometry of the right nasal cavity.

then Figure 3 shows that at radius 2 during inspiration of the right nasal cavity,

$$R_2 = \frac{\Delta p_2}{\mathring{V}^n}$$

It is agreed that for the radius 2 ($\mathring{V} = 0.2$ l/sec), the value of n is approximately equal to 1 (Melon, 1973; Clément, 1978). The relation between the pressure gradient scale is 1 to 10, so that:

$$\tan v_2 = \frac{\Delta p_2}{10\ \mathring{V}_2} \text{ or } R = \tan v_2 \text{ (Figure 3).}$$

For example, in a total nose with a radius 2 upon inspiration:

$$R_2 = \frac{\Delta p_2}{5\ \mathring{V}_2} = 5 \tan v_2,$$

because the relationship between the pressure gradient scale and the flow scale is 1 to 5.

In clinical use, v_0 and R_0 can be calculated, which for inspiration and expiration yields equal values. However, from experience it has been found that numerous people have more difficulties with inspiration than expiration (hence, for physio-

logical reasons, it is more meaningful to calculate R farther from the origin and for both inspiration and expiration. It was also seen that patients with a septal deviation have a curvature that is greater than that of normal patients.
The degree of curvature is given by the factor c formula 1) so than the distinction between a normal test subject and patients with a septal deviation is better made further from the origin, for example at radius 2.

CURRENT RESEARCH

The mathematical model was tested with a sample selected by the same E.N.T. specialist from a population of healthy, young people without nose complaints. From this group, all subjects were rejected who had mild septal deviations or any abnormality of the nasal mucosa.
This ultimately lead to the rejection of 80% of the subjects. In the end, 32 subjects remained: 11 men and 21 woman ranging in ages from 18 to 30. The same researcher performed an anterior rhinomanometry on all subjects and succeeded in performing a posterior rhinomanometry on half of the subjects. On all measurements, the following statistical manipulation were done:

1. Using the mathematical model, for each nose side v_0, c_E, c_I; v_{2I}, v_{2E}, R_E, R_I and R_{total} were calculated. For all these values the average and standard deviation were also derived. A comparison of the values was undertaken to determine whether a normal distribution existed both for the right and left nasal cavity. Correlations were sought between these values as well.
2. The validity of the mathematical model was tested by a comparison of values v_{2E} and v_{2I} calculated from the mathematical model and from the actual recording.
3. Expiration and inspiration of the right nose side were followed to elucidate any significant difference between men and woman concerning the average value and the standard deviation of v_2 as well for expiration as for inspiration of the right nose side.
4. The mathematical model of the total nose was used to derive the total nose resistence. The total nose resistence was also calculated using the direct recordings of posterior rhinomanometry. Both values were compared.

RESULTS

1. For the right nose side, table II gives the most parameters derived from the observations of the following values:
   - v_0 this is the angle v_0 of the model $v(r) = v_0 + c \cdot r$
   - c_E this is the parameter c of the mathematical model (1) for expiration
   - c_I this is the parameter c of the mathematical model (1) for inspiration

v_{2E} this is the angle $v(2) = v_0 + c_E \cdot 2$: this angle is of great importance as it is related to the clinical nose resistence R_E by: $R_E = 10 \tan v_{2E}$

v_{2I} idem as for v_{2E}

R_E nose resistence at radius 2 for expiration derived from the mathematical model

R_I idem as R_E

R_{tot} $R_{total} = \frac{R_E + R_I}{2}$

The parameter, b_1 known as the coefficient of asymmetry, is a measure of the degree in which the studied variable has a gaussian distribution (for the normally distributed value this parameter is equal to 0): as this value deviates from 0, the phenomenon becomes progressively less normally distributed.
In similar fashion, the parameter b_2 (coefficient of kurtosis) is a measure of normality: the closer this parameter is to 3, the more normally distributed the studied variabels are.
According to the chi-square test for goodness of fit as well as the hypotheses tests on b_1 and b_2 the normality hypothesis is accepted for all studied values. This conclusion permits us to assert that, for example 95% of the non-pathological cases have a mathematical model calculated nose resistance which is less than $2.1 + 1.64\ (0{,}6) = 3.21$. This also permits us to test a hypothesis with relatively small sample whereas in the case of non-normality a much larger number of subjects is needed.
Table 3 gives the correlation between the different values.
The closer the correlation coefficients is to unity, the greater the degree of relationship between the two variables. For example, a correlation coefficient of 0,24 between v_0 and c_E indicates that there is practically no correlation between the two values.
The correlation coefficient of 0,83 between c_I and c_E on the contrary, indicates that a person with low value of parameter c for inspiration has a high probability of having a low value c for expiration (at least for the non-pathological cases studied herein).
In this sence, correlation coefficients of 0.91 for v_{2I} and v_{2E} and 0.88 for R_E and R_I signifies that the mathematical model possesses the ability to predict that a test subject with a high nose resistance upon expiration will also have a high nose resistance upon inspiration.
This is a useful property in a model in light of the fact that the property also exists in reality for non-pathological test subjects.
Table 4 and Table 5 gives the same parameters as Tables 2 and 3 but for the left nasal cavity. One sees , however, that the parameters b_1 for all studied values is more deviant from the normal zero value and that b_2 is more deviant from the

Table 2. Right nasal cavity.

	v_0	C_E	C_I	v_{2E}	v_{2I}	R_E	R_I	R_T
Mean	3.13	4.14	3.90	12.05	11.55	2.14	2.06	2.1
Variance s^2	3.88	1.78	2.09	13.48	15.62	0.45	0.51	0.47
Standard deviation	1.97	1.33	1.45	3.67	3.95	0.67	0.72	0.68
b_1	0.34	0.003	0.57			0.22	0.23	0.25
b_2	2.58	2.44	3.36			2.43	2.20	2.17

Table 3.

	v_0	C_E	C_I	v_{2E}	v_{2I}	R_E	R_I	R_T
v_0	–	0.24	0.30	0.71	0.72	–	–	–
C_E	0.24	–	0.83	–	–	–	–	–
C_I	0.30	0.83	–	–	–	–	–	–
v_{2E}	0.71	–	–	–	0.91	–	–	–
v_{2I}	0.72	–	–	0.91	–	–	–	–
R_E	–	–	–	–	–	–	0.88	0.98
R_I	–	–	–	–	–	0.88	–	0.96
R_T	–	–	–	–	–	0.98	0.96	–

Table 4. Left nasal cavity

	v_0	C_E	C_I	v_{2E}	v_{2I}	R_E	R_I	R_T
Mean	4.94	4.17	4.40	13.28	13.75	2.40	2.50	2.45
Variance s^2	13.48	4.65	7.12	36.50	47.64	1.32	1.80	1.51
Standard deviation	3.67	2.15	2.67	6.04	6.90	1.15	1.34	1.23
b_1	1.44	1.18	1.78			0.98	3.17	1.08
b_2	4.52	3.78	5.48			2.33	3.17	2.70

Table 5.

	v_0	C_E	C_I	v_{2E}	v_{2I}	R_E	R_I	R_T
v_0	–	0.14	0.15	0.71	0.64	–	–	–
C_E	0.14	–	0.93	–	–	–	–	–
C_I	0.15	0.43	–	–	–	–	–	–
v_{2E}	0.71	–	–	–	0.95	–	–	–
v_{2I}	0.64	–	–	0.95	–	–	–	–
R_E	–	–	–	–	–	–	0.94	0.97
R_I	–	–	–	–	–	0.97	0.98	–

normal value of three. From a statistical analysis by the chi-square test for goodness of fit and the hypothesis tests on b_1 and b_2 it appears that the normality hypothesis must be rejected. The existence of a normal distribution for the values of the right and not of the left nasal cavity can be explained in different fashions.

One possible explanation is the fact that the test subjects were strictly selected to exclude pathological cases.

The selection is, however, not entirely objective and could have erroneously included pathological cases, which would strongly disturb the results. Another less probable explanation is that medical reasons exist for the broader distribution of the left in comparison to the right nasal side.

2. To ascertain the value of the mathematical model for clinical use, the angle of the recording with the $\mathring{V}$ axis on radius 2, thus v_{2E} and v_{2I} was calculated from the mathematical model and compared with the same angles as directly measured on the recording. These angles are of great importance since the resistance corresponds to 10 tan v_2.
   For expiration, the average difference between the v_{2E} (model) and the angle v_{2E} (recording) is equal to −0.42 degrees with a standard deviation of 1 degree. From this, it can be derived that for resistance greater than 2 mBar sec/l, the difference between model and real resistance amounts to maxillary 20%, which indicates the quality for the model for such resistances. For lower nose resistances (1) the adaption of the model is less acceptable; the difference between model and recorded nose resistance can amount to 50%. For inspiration, the average difference between model and recorded v_{2I} is equal to 0.3 degrees with a standard deviation of 1 degree. With respect to resistance, the identical results as in expiration were obtained.
3. The expiration and the inspiration of the right nose side was examined to determine if there was a significant difference in the average and the standard deviation of v_2 for the male and female populations.
   It appears that, for all levels of significance smaller than 20% there is no significant difference in the standard deviation of v_2 or in the average of v_2.
4. Broms et al. (1979) offered a method of constructing a model of the total nose departing from the models of the left and right nose side. His group utilized a hypothesis that on first sight is very logical; at a specific pressure, the total flow is the sum of the flows for the right and left nasal cavity (mass balance hypothesis for the total nose). The total nose resistance at radius 2 for inspiration and expiration can be calculated from the total nose model. R_{ET} and R_{IT} are these values. If a posterior rhinomanometry is performed, then the total nose resistance can also be calculated using the mathematical model for one nasal cavity as previously expounded. These are R'_{ET} and R'_{IT}. It appears that the average for the difference between R'_{ET} R_{ET} is 0.09 with a standard deviation of 0.41.
   This means that the deviation in more than 90% of the cases is less than $(0.09)+2(0.41)=0.9$ mBar sec/l. This means however, that for resistances at radius 2 the differences can be equal to 50% of the actual nose resistance. These differences can be due to the fact than the mass balance hypothesis for the total nose is, in some cases, only a coarse approximation for the physical reality. Another possibility is that posterior rhinomanometry as currently per-

formed does not yield a very reliable registration.

In any case, it seems that further research on the value of the mass balance hypothesis and the value of posterior rhinomanometry is necessary and that the results of the model of the total nose must be interpreted with caution.

CONCLUSIONS

The following conclusions can be drawn from the statistical viewpoint:

1. The derived mathematical model values for one nose side v_0, c etc. show a favourable behaviour concerning average distribution and dependency. The non-normality of the different values for the left nasal side makes further research necessary but rhinomanometry combined with the mathematical model is a valuable tool in the statistical examination of nose resistance.
2. For nose resistance greater than 1 mBar sec/l, the values v_0, $c_E$and c_I contain enough information to describe the recording numerically.
3. From the results, it was obvious that for a flow of 0.2 l/sec no significant difference was found in nose resistance with respect to distribution for males in comparison with females.
   This is the reasoning for making no further distinction between these two groups in subsequent research.
4. The mathematical model described by Broms et al. (1979), for the total nose, derived from the data for each nose side must be carefully applied. A more rigorous examination is also necessary into mass balance hypothesis of the total nose.

DISCUSSION

The values of the total nose resistance per nose side and for both sides as calculated with the mathematical model agree rather well with the values given in the literature (Melon et al., 1979). This indicates the utility of the model. The normal distribution of the different values was not shown with other mathematical models. The authors believe that this is the best mathematical model now in existence.

The calculation of the nose resistance utilizes the law of Poisseuille which is not completely satisfactory. Since the curvature of the line expressed by value c is related to the exponent "n" of the formula:

$$R = \frac{p}{\mathring{V}^n}$$

in the future, it must be included in the calculation of R.

RÉSUMÉ

Les auteurs commentent les modèles mathématiques proposés par un groupe Suédois (Broms et al.). Ce modèle permet d'appliquer une équation mathématique, une régistration du gradient de pression et de débit que l'on obtient au cours d'une rhinomanométrie antérieure ou postérieure active. Ce modèle mathématique a été mis en application chez 32 sujets normaux, ceux-ci afin d'évaluer sont application mathématique, statistique et clinique. Les auteurs sont convaincus qu'il s'agit du meilleur modèle mathématique existant actuellement bien qu'il ne soit pas dénué de critique.

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